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Probability & Statistics
Assignment - 2

Ans 1. Let X_i be claim size ~~and~~ of home insuran
Y_i be claim size of car insurance

$\therefore X \sim N(800, 100^2)$
$Y \sim N(1200, 300^2)$

$P(Y_1 + Y_2 + Y_3 > X_1 + X_2 + X_3 + X_4 + 800)$

$P(Y_1 + Y_2 + Y_3 - (X_1 + X_2 + X_3 + X_4) > 800)$

$(Y_1 + Y_2 + Y_3) - (X_1 + X_2 + X_3 + X_4) \sim N(3 \times 1200 - 4 \times 800,$
$300^2(3) + 100^2(4)) \sim N(400, 310000)$

$P(Z > 0.71842) \Rightarrow 1 - P(Z < 0.71842)$

$= 1 - P(Z < 0.72)$
$= 1 - 0.76424$
$= 0.23576$

Ans. $N \sim$ ~~B~~ Neg. Binomial (k, p)

$$P(N=n) = \cancel{n+2}\ {}^{n+2}C_n (0.9)^3 (0.1)^n$$

$$= \cancel{(n+3)-1}\ {}^{n+3-1}C_{n-1} (0.9)^3 (0.1)^n$$

So, $k = 3$

$p = 0.9$

$$M_y t = M_n (\log M_x t)$$

$X \sim$ Gamma $(6, 2)$

$Y \to$ Total Claim amount

MGF of Y

$$M_y(t) = E(E(e^{yt}/N=n))$$

$$E(e^{yt}/N=n) = E(e^{x_1 t + x_2 t \cdots x_n t})$$

$$= E(e^{tx_1}) \cdot E(e^{tx_2}) \cdots E(e^{tx_n})$$

$$= \left(1 - \frac{t}{2}\right)^{-6n}$$

So, $E(E(e^{yt}/N=n)) = E\left(\left(1 - \frac{t}{2}\right)^{-6n}\right)$

$$= E\left(e^{n \log(1 - t/2)^{-6}}\right)$$

$$= M_N\left(\log\left(1 - \frac{t}{2}\right)^{-6}\right)$$

$$M_N(t) = \left(\frac{pe^t}{1-qe^t}\right)^k$$

$$\Rightarrow \left[\frac{pe^{\log(1-t/2)^{-6}}}{1-qe^{\log(1-t/2)^{-6}}}\right]^k$$

$$\Rightarrow \left[\frac{p(1-t/2)^{-6}}{1-q(1-t/2)^{-6}}\right]^3$$

$$\Rightarrow \left[\frac{(0.9)\left(1-\frac{t}{2}\right)^{-6}}{1-0.1\left(1-\frac{t}{2}\right)^{-6}}\right]^3$$

$$V(Y) = E(N)\,V(X) + (E(X))^2 \cdot V(N)$$

$$\Rightarrow \left[\frac{3}{0.9} \times \frac{6}{4}\right] + \left(\frac{6}{2}\right)^2 \times \frac{3(0.1)}{(0.9)^2}$$

Standard Deviation $= 2.887$

Ans 3. i) $f(x) = \lambda e^{-\lambda(x-\alpha)}$

$$MGF = E(e^{tx})$$

$$= \int_{\alpha}^{\infty} e^{tx} \lambda e^{-\lambda(x-\alpha)} dx$$

$$M_x(t) = \lambda \int_{\alpha}^{\infty} e^{tx} e^{-\lambda x} e^{\lambda\alpha} dx \; dx$$

$$= e^{\lambda\alpha} \int_{\alpha}^{\infty} e^{-(\lambda-t)x} dx$$

$$\Rightarrow \frac{e^{\alpha\lambda}\lambda}{t-\lambda}\left[e^{-(\lambda-t)x}\right]_{\alpha}^{\infty}$$

$$\Rightarrow \frac{e^{\lambda\alpha}\lambda}{t-\lambda}\left[-e^{-(\lambda-t)\alpha}\right]$$

$$\Rightarrow e^{t\alpha}\frac{\lambda}{\lambda-t}$$

(ii) $M_x t = \frac{\lambda e^{t\alpha}}{\lambda-t} = \lambda e^{t\alpha}(\lambda-t)^{-1}$

$$M_x'(t) = \lambda\left[e^{t\alpha}(\lambda-t)^{-2}(1) + (\lambda-t)^{-1} e^{t\alpha}(\alpha)\right]$$

$$= \lambda e^{t\alpha}(\lambda-t)^{-2}\left[\alpha(\lambda-t)+1\right].$$

put $[t=0]$ $= \lambda(\lambda)^{-2}\left[\alpha(\lambda)+1\right]$

$$= \frac{1}{\lambda}\left[\lambda\alpha+1\right] = \frac{\lambda\alpha+1}{\lambda}$$

$$E(X)^2 \quad M''_X(t) = \alpha\left[\alpha e^{t\alpha}(\alpha-t)^{-2} + \alpha^2(\alpha-t)^{-1}e^{t\alpha} + e^{t\alpha}(-2)(\alpha-t)^{-3}\alpha + (\alpha-t)^{-2}e^{t\alpha}\right]$$

put $t=0$

$$\Rightarrow \frac{2\alpha}{\alpha} + \alpha^2 + \frac{2}{\alpha^2}$$

$$V(X) = E(X)^2 - [E(X)]^2$$

$$= \frac{2\alpha}{\alpha} + \alpha^2 + \frac{2}{\alpha^2} - \left[\alpha + \frac{1}{\alpha}\right]^2$$

$$= \frac{1}{\alpha^2}$$

Ans 4.

$$G_V(t) = \left(\frac{p}{1-qt}\right)^k$$

$$G_V(t) = \sum t^r \times P(V=r)$$

$$P(V=4) = \frac{\Gamma k+4}{\Gamma 5 \, \Gamma k} p^t q^k$$

$$= \frac{(k+3)!}{(k-1)!\,4!} p^k (1-p)^k$$

Ans 5.

$$f(x,y) = ax(x+y)$$

$$P\left(Y > \frac{1}{3}X + 10\right)$$

$$\Rightarrow a\int_{10}^{20}\int_{0}^{5} (x^2+xy)\,dx\,dy = 1$$

$$\Rightarrow a\int_{10}^{20} \left(\frac{x^3}{3} + \frac{x^2}{2}y\right)_0^5 dy$$

$$\Rightarrow \left(\frac{125}{3}y + \frac{25}{4}y^2\right)_{10}^{20} = \frac{1}{a}$$

$$\Rightarrow \cancel{8.3}\ 833.33 + 2500 - 416.667 + \cancel{0.5} = \frac{1}{a}$$

$$a = 0.00043636.$$

(iii) $f_{Y/X} = \dfrac{f(x,y)}{f(x)}$

$$f(x) = \frac{3}{6875}\int_{10}^{20} (x^2 + xy)\,dy$$

$$= \frac{30x}{6875}(x+15)$$

$$f_{Y/X} = \frac{3}{6875}\,\frac{x(x+y)}{30x} \times 6875(x+15)$$

$$\frac{3}{6875}\,\frac{x(x+y)}{10x}\,\frac{6875}{(x+15)}$$

$$\text{So } E(Y/X) = \frac{1}{x+15}\left[15x + \frac{700}{3}\right]$$

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Sol 6. $X \sim Poi(\lambda)$ $Y \sim Poi(\mu)$

$M_X(t) = e^{\lambda(e^t-1)}$

$M_Y(t) = e^{\mu(e^t-1)}$

So $Z = X - Y$

$M_Z(t) = E(e^{tz}) = E(e^{t(x-y)})$

$= E\left(e^{tx} / e^{ty}\right)$

$= \dfrac{e^{\lambda(e^t-1)}}{e^{\mu(e^t-1)}}$

$= e^{(\lambda-\mu)(e^t-1)}$

By ~~the~~ uniqueness of MGF of z

$Z \sim Poi(\lambda - \mu)$

Now let $Z = X + Y$

$E(e^{tz}) = e^{(\lambda+\mu)(e^t-1)}$

Therefore, similarly by uniqueness of z

~~z Poi~~ $Z \sim Poi(\lambda + \mu)$

Ans 7

		X			
		1	2	3	4
Y	1	0.1	0.1	0.05	0.15
	2	0	0.3	0.1	0.2

$$V(X/Y=2) = E(X^2/Y=2) - (E(X/Y=2))^2$$

$$E(X^2/Y=2) = \sum x^2 \frac{P(X=x, Y=2)}{P(Y=2)}$$

$$= \frac{1(0)}{0.6} + \frac{4(0.3)}{0.6} + \frac{9(0.1)}{0.6} + \frac{16(0.2)}{0.6}$$

$$= 8.8333$$

$$E(X|Y=2) = \sum \frac{xP(X=x, Y=2)}{P(Y=2)}$$

$$= 2.83333$$

$$V(X/Y=2) = 0.807$$

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(iii) $f(u,v) = \frac{48}{67}(2uv - v^2)$

$E(U/V=v) \Rightarrow$

$\frac{f(u,v)}{f(v)} = f(U/V=v)$

$f_v = \frac{48}{67}\int_0^1 (2uv - v^2)\,du$

$= \frac{48}{67}\left[2u^2v - \frac{v^3}{3}\right]_0^1$

$= \frac{48}{67}\left[v - \frac{1}{3}\right] = \frac{16}{67}(3v-1)$

$f(v) = \frac{16}{67}(3v-1)$

$\frac{f(u,v)}{f(v)} = \frac{48}{67}\left(\frac{2uv^2 - v^2}{16(3v-1)}\right) \times 67$

$= 2u^2v^2 - v^3$

Now $E(U/V=v) = \frac{3}{3v-1}\int_0 u(2uv^2 - v^2)\,du$

$= \frac{8v^2 - 3}{4(3v-1)}$

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Ans 8. $X \sim Gamma(3,2)$

$E(X) = 3/2$

$V(X) = 3/4$

$E(Y/X=x) = 3x+1$

$Var(Y/X=x) = 2x^2+5$

$E(Y) = E(E(Y \mid X=x))$

$= E(3x+1) = E(3x) + E(1)$

$= 3E(x) + 1$

$= 4$

$V(Y) = E(V(X \mid Y)) + V(E(X \mid Y))$

$= E(2x^2+5) + V(3x+1)$

$= 2E(X^2) + 5 + 9V(X) + 0$

$V(X) = E(X^2) - [E(X)]^2$

$E(X^2) = V(X) + [E(X)]^2$

$= 13/2$

$\therefore$

$V(Y) = 2(3) + 5 + 9/3/4$

$= \frac{71}{4}$ or 17.75

Standard Deviation of $Y = \sqrt{71/4}$

$$= \frac{\sqrt{71}}{2}$$

Ans 9:

$E(X) = 3 \qquad V(X) = 4$

$E(Y) = 4 \qquad V(Y) = 1$

$r = 0.3 = \frac{Cov(X,Y)}{\sqrt{V(X) \cdot V(Y)}} = \frac{Cov(X,Y)}{\sqrt{(4)(1)}}$

$Cov(X,Y) = -0.6$

$Z = X + Y$

(i) $Cov(X, Z) = Cov(X, X+Y)$

$= Cov(X,X) + Cov(X,Y)$

$= V(X) + Cov(X,Y)$

$= 4 + (-0.6)$

$= 3.4$

(ii) $Var(Z) = Var(X+Y) = Cov(X+Y, X+Y)$

$= Cov(X,X) + Cov(X,Y) + Cov(Y,Y) + Cov(Y,X)$

$= V(X) + V(Y) + 2Cov(X,Y)$

$= 4 + 1 + (-0.6)$

$Var(Z) = 4.4$

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Ans/10. $f(x,y) = Kx^{-\alpha} e^{-y/\beta}$

$$\Rightarrow K \int_1^{\infty} \int_1^{\infty} x^{-\alpha} e^{-y/\beta} \, dx \, dy = 1$$

$$\Rightarrow K \int_1^{\infty} e^{-y/\beta} \left[\frac{x^{-(\alpha-1)}}{-(\alpha-1)} \right]_1^{\infty} dy = 1$$

$$\Rightarrow \frac{K}{1-\alpha} \int_1^{\infty} e^{-y/\beta} (-1) \, dy = 1$$

$$\Rightarrow \cancel{\frac{-K}{1-\alpha} \int_1^{\infty}} \quad \frac{-K}{1-\alpha} \left[\frac{e^{-y/\beta}}{-y/\beta} \right]_1^{\infty} = 1$$

$$\Rightarrow \frac{-K\beta}{1-\alpha} \left(e^{-\infty} - e^{-y/\beta} \right) = 1$$

$$\Rightarrow \frac{-K\beta}{1-\alpha} \left(-a + e^{-y/\beta} \right) = 1$$

$$K = \frac{\alpha - 1}{\beta \left(e^{-y/\beta} \right)}$$

Hence proved